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Accuracy in the detemination of fusion barriers

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Abstract. Although there is no proof of the validity of the method, in a lot of papers, one computes the Second derivative of the the experimental fusion cross section multiplied by the energy to extract the fusion barriers directly from the experiment. The purpose of this paper is to check the consistency of the method, as experimental points are sandwiched inside their error bars. We therefore analysed the validity, by giving ourselves a cross section σ(E), resulting from the coupled channel calculation code ECIS, upon which, in spite of the complexity of the calculations, we have full control. We took these calculated points as the "experimental" ones and we altered them by multiplication by small random numbers. This is intended to simulate the error bars, of which we want to examine the influence on the fusion barriers. We find that in spite of the rough predictions yielded by the second derivative method, this task requires data with a precision difficult to reach. Furthermore, a careful check of the predictions of this method for coupled channels calculations shows that, due to the errors bars, this approach adds spurious results.

1 Introduction

When two nuclei a and A collide they may form a third one C* such that fusion occurs

$$a + A \Longrightarrow C^* \Longrightarrow b + B^*$$

The incident energy E permits to subdivide the formation of the system C* in two regimes, one under and one above the repulsive potential barrier between the two nuclei. Above the barrier, the incident energy is high enough to overcome the repulsion, the nuclei collide and then undergo a decay. Under the barrier, the reaction is classically forbidden, but they may tunnel through the barrier and fuse.

The problem is to find this barrier, and/or, if several barriers are present, how they are distributed. It is important to determine the barrier(s) limiting the two energy regimes, as they govern the fusion cross section, the obtention of new species of nuclei etc., but barriers are not directly observable and one needs to develop indirect methods to get them. Accordingly, a lot of work [1–4], have been devoted to these questions, impulsed by an analytic expression for the fusion cross section, valid above and under the barrier, derived in 1973 by Wong [5]. To locate the barriers, Rowley and collaborators [6] presented a method which consists in associating the barriers to the peaks of the second derivative with respect to E, of E times the fusion cross section. They add that their method, when using the experimental fusion cross sections data, will lead directly to the barriers.

Our purpose was to analyse the validity of this claim which seemed reasonable although needing a clear proof. We show [7] however that due to the unavoidable error bars, very high precision data are required in order to extract fusion barriers, therefore leaving open the question of confidence in the method.

2 The second derivative approach

In a purely classical way, Weisskopf [8] in 1937, derived an expression for the fusion cross section σ(E) valid when the incoming energy E is much greater than the height of the barrier B

$$\sigma(E) = \frac{\pi R^2}{E}(E - B) \qquad E \gg B \tag{1}$$

R is the position of the barrier *i.e.* at R the total (nuclear plus Coulomb) scattering potential takes a value

$$B = V(R).$$

For a quantal system, the probability for compound nucleus formation, *i.e.* the fusion cross section is

$$\sigma(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) T_l(E) \tag{2}$$

where $T_l(E)$ is the fusion probability for the l-th partial wave and $k^2 = 2\mu E/\hbar^2$ with μ the reduced mass of the system.

Wong [5] introduced the following approximations in order to obtain $\sigma(E)$:

i) Around the top R, one may approximate V(r) by its Taylor expansion, and replace the total interaction by the following parabola

$$V(r) \approx B - \frac{\mu\omega^2}{2}(r-R)^2$$

with curvature

$$\omega^2 = -\frac{V''}{\mu}\Big|_R > 0.$$

Adding the centrifugal barrier, V(r) becomes $V_l(r)$ such that

$$V_l(r) \approx B - \frac{\mu\omega^2}{2}(r-R)^2 + \frac{\hbar^2}{2\mu r^2} l(l+1) \qquad (3)$$

with the only l-dependence coming only from that last term since Wong [5] has shown that R and ω are insensitive to l .

ii) Moreover, for $T_l(E)$ Wong [5] takes the expression derived by Hill and Wheeler [9]

$$T_l(E) = \Big[1 + \exp\Big(\frac{2\pi}{\hbar\omega}(V_l(R) - E)\Big)\Big]^{-1} \qquad (4)$$

and replaces the discrete sum over l in (2) by an integral. This leads to the well known Wong formula describing approximately the fusion cross section for any E. It generalize (1) and extend its range of validity around and, to some extent, under the barrier.

$$\sigma(E) = \frac{\pi R^2}{E}\frac{\hbar\omega}{2\pi}\ln\Big[1 + \exp\Big(\frac{2\pi}{\hbar\omega}(E-B)\Big)\Big] \qquad (5)$$

iii) Far beyond the barrier, expression (5) gives back (1)

$$\sigma(E) = \frac{\pi R^2}{E}(E-B) \qquad E \gg B$$

and under the barrier we get

$$\sigma(E) = \frac{\pi R^2}{E}\frac{\hbar\omega}{2\pi}\exp\Big(\frac{2\pi}{\hbar\omega}(E-B)\Big) \qquad E \ll B. \qquad (6)$$

It is easy to analyse the properties of (5). In particular, noticing that the second derivative of (1) is

$$\frac{d^2(E\sigma)}{dE^2} = \pi R^2 \delta(E-B) \qquad (7)$$

Rowley et al. [6] using (5) generalized it to

$$\frac{d^2(E\sigma)}{dE^2} = \pi R^2 \frac{2\pi}{\hbar\omega}\frac{e^x}{(1+e^x)^2} \qquad (8)$$

with $x = (2\pi/\hbar\omega)(E-B)$. These two last expressions, both strongly peaked around $E = B$, are formally equivalent.

For the $^{32}S + ^{64}Ni$ system, they found [6] a good agreement between the expression (8) and an optical model calculation. However, Dasso [10] pointed out that the approximate character of this type of method imposes limits on the possibility to extract precise nuclear structure information.

3 Accuracy of the method

In their paper, Rowley et al. [6], claim that barriers can be obtained *via* the second derivative of the set of relevant experimental points. On one hand, the numerical computation of derivatives [11] may already be rather imprecise, even for well-behaving analytical functions. On the other hand, the experimental points are given with their errors bars. In other words a dot has some probability to be found sandwiched anywhere inside its error bar.

To check the sensitivity of the second derivative method, we took a cross section resulting from the coupled channel calculation code ECIS [12] upon which, in spite of the complexity of the calculations, we have full control, namely where the fusion barriers are, what are the states participating to the interaction (as we introduce them in the code) etc. This procedure allows us to know the sub-Coulomb fusion barriers with a very good accuracy: we know our input and know what must be the output.

We computed the reaction cross section $\sigma_{\rm fus}$, for the $^{32}S + ^{24}Mg$ system, using Raynal [12] code ECIS with the optical potential parameters given by Rhoades-Brown et al. [13]. Therefore we knew where the fusion barrier was expected to lie, as the parameters used by these authors originate from the empirical formulas proposed by Broglia and Winter [14]. In our calculations the only channel present is fusion since all the flux whose energy is higher than the barrier is absorbed [15]. Indeed, the second derivative of $E.\sigma_{\rm fus}$, shows a peak (solid line in Fig. 1) that we *interpret* as a barrier, at an energy in full agreement with what was expected from our input. This is also along the result of Rowley et al. [6] mentioned earlier on their $^{32}S + ^{64}Ni$ system computations.

To take into account the error bars on our "experimental" data, we then modified the ECIS code output $\sigma_{\rm fus}$ by a small percentage into

$$\sigma = \sigma_{fus}(1 + r/K) \qquad (9)$$

Here r is a random number between -0.1 and $+0.1$ and K will be defined shortly. Obviously, the resulting cross section σ will not exhibit the smooth shape of $\sigma_{\rm fus}$. The random shift of the "dots" inside their error bars was a way to represent the *uncertainty* linked to the measure and a way to see if and how the peaks move as (it is claimed that peaks correspond to barriers).

We compute now $(E.\sigma_{\rm fus})''$ and $(E.\sigma)''$. The results are displayed in the left hand side of Fig. 1, where the solid line represents $(E.\sigma_{\rm fus})''$. We see that a *very small* alteration of $\sigma_{\rm fus}$, induces a chaotic behavior on $(E.\sigma)''$ (dots) at higher energies. The effect of the noise being overwhelming at the higher energies, we have attenuated its magnitude by the extra factor 1/K, K = 10, to render the results presentable (ansatz 1 on Fig. 1). Note that with K = 10, the alteration is smaller than 1%. To regularize a little more σ we modified (9) by transforming K into K_E such that

$$\sigma = \sigma_{fus}(1 + r/K_E) \qquad (10)$$

where K_E is proportional to the incident energy. As well known, for a constant acquisition time, the higher the en-

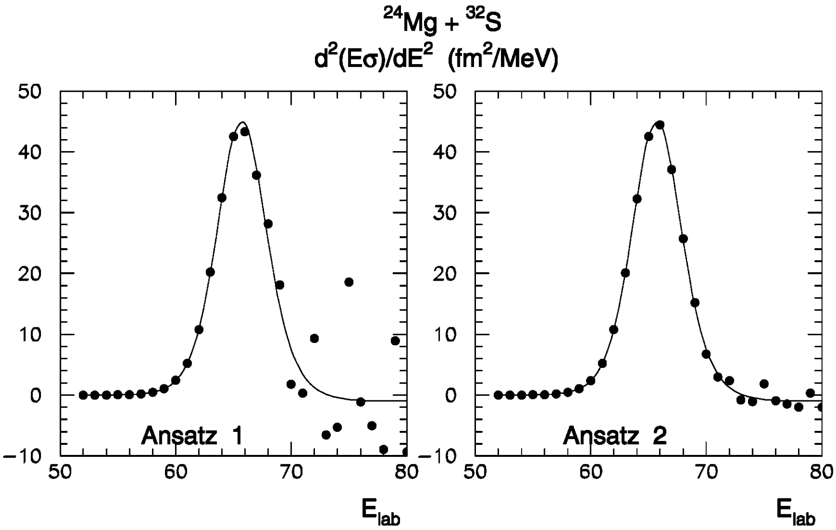

Fig. 1. Comparison of the two ansatz described in (9) and (10) respectively to obtain the barrier with a pseudo-experimental cross section. The solid line and the dots represents $(E.\sigma_{\rm fus})''$ and $(E.\sigma)''$ respectively

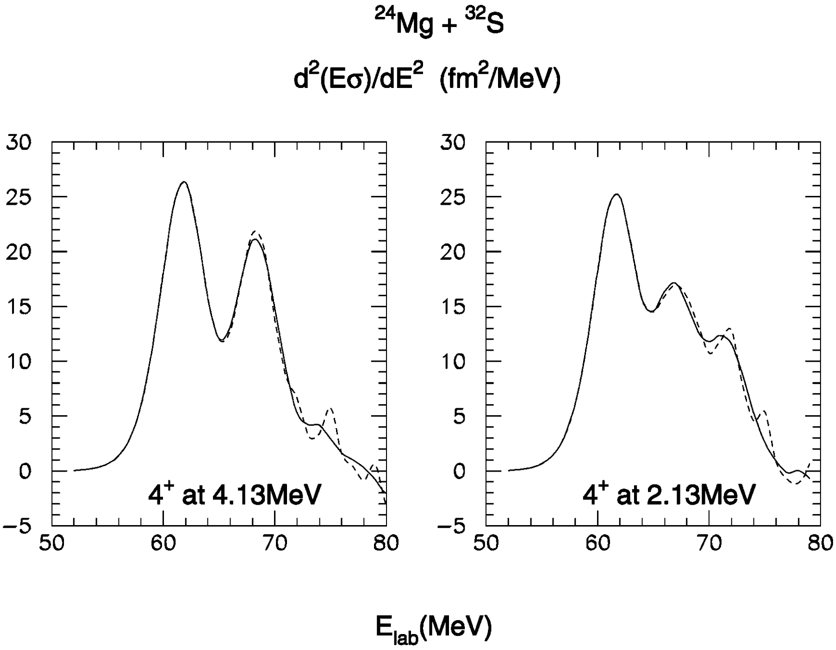

Fig. 2. Results of calculations taking into account the excited states of ^{24}Mg (left) and with a modified value for the 4^+ (right)

ergy the smaller the error bar. We see (right hand side of Fig. 1) that the overall result is improved. In fact, the ansatz of 10 (ansatz 2 on Fig. 1), kills almost all the noise and smoothens the cross sections especially at the higher energies.

The striking feature is that the peak remains in any of the two altered situations. One is then tempted to argue in favor of the method. But, in fact, the agreement, overall before the peak, is an artefact. Although we are dealing with cross sections obtained from a coupled channel code, the analogy with the Wong formula will explain the situation. At $E << B$, the decrease of $(E\sigma)''$, is governed by the decrease of the exponential present in (6). This cannot be balanced by the linear increase of E. In other words, the exponential term will make $(E\sigma)'' \approx 0$ as long as $E - B$ is negative enough. Around $E - B$, (6) presents a peak and then, for $E - B > 0$, we obtain just the cahotic behavior of Fig. 1 due to the difficulty to obtain accurately the derivative of that exponentially increasing function. This have been also outlined by Dasso [10].

4 Influence of the excited states

Taking up the second derivative method [6], we have investigated the influence of the coupling to the lowest excited states of ^{24}Mg scattering with ^{32}S. Again we get $\sigma_{\rm fus}$

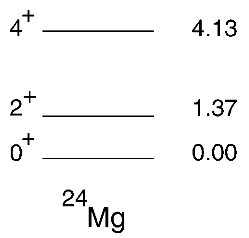

Fig. 3. The spectrum of ^{24}Mg used for ECIS calculations

from the Raynal [12] code ECIS [15] with :

$$\sigma_{fus} = \sigma_{react} - \sum \sigma_{inel}$$

The left hand side of Fig. 2 shows the unmodified (solid line) and smoothed (dashed curve) computations of $(\mathrm{E}.\sigma_{\mathrm{fus}})''$ and $(\mathrm{E}.\sigma)''$ respectively. Here by smoothed we mean dividing by the $\mathrm{K_E}$ factor like in (10). We see on this figure

i) three bumps at respectively 62 MeV, 68 MeV and 74 MeV occuring when using the ^{24}Mg low-energy spectrum of Fig. 3 ,

ii) a series of bumps, some bearing a physical meaning and some generated by the noise.

To examine the nature of the very small bump that occurs on the solid line at 74 MeV, we have lowered the 4^+ state at the artificial energy of 2.13 MeV, instead of 4.13 MeV. The resulting peaks are displayed on the right hand side of Fig. 2. While the peak at 62 MeV remains rather steady, there is some interplay between the peaks at 68 and 74 MeV. On the other hand, a very small noise, indeed, introduces some unphysical peaks (dashed lines). As their positions are also perturbed by the modification of the energy of the 4^+ level, when we perturbate the physical ones, we have no way to drop off the spurious bumps, outlining the difficulty to make any clear cut interpretation.

5 Conclusion

As a conclusion, this second derivative method analyzed here, leaves pending some indeterminations. There is no justification for the procedure. Although giving rough predictions, the relation between the peaks and the coupled channels calculations is more intricate than expected. Actually, the extraction of barriers directly from experiments requires data with a precision ($< 1\%$) difficult to obtain, outlining the need for more accurate methods to determine the barriers. Finally, here we have neither statistical nore systematic errors. The only errors present here are just the ones generated on purpose by the random numbers.

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